Measures of Market Return

FinanChi, September 2024

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Measures of Returns

Measures of market return are useful in several ways, most saliently in the Capital Asset Pricing Model (CAPM). While the US markets are of most interest to many investors, and the S&P 500 index remains the most widely used measure of market return, other measures are used both in the US and internationally, such as the Wilshire 5000 and the MSCI.

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Quality

- Different measures have different pros and cons. In particular, at FinanChi, we use multiple measures in our modeling and wish to rely on a formal definition of market return that is well-defined, from a mathematical perspective, and can broadly encompass the universe of possible measures of market return.
- Existing measures of market return have varying levels of quality. We can categorize those as follows: Relevance, Recency, Volatility, and Decomposition.

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Relevance and Decomposability

A measure may work well but lack relevance if it cannot be used in models. The S&P 500 is quite relevant, because it directly measures actual returns on any time scale one might choose.

Book value movement should correlate to market returns, but is not relevant to most investors, other than as a data point for decision making.

The S&P 500 can be quite volatile, but using moving averages easily solves that problem.

Decomposition is a major flaw with existing market return measures. While earnings or stock prices tell you exactly how well a firm is doing, there are not many ways to decompose the number into different factors. While we can decompose the S&P 500 returns by sector, we cannot decompose it into returns associated with different management activities or market forces, such as supplier pricing, or marketing efficacy.

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Solution

To solve this problem, the most basic requirement is to have a definition of market return.

This requires placing constraints on what constitutes such a measure. We group those constraints as follows: Mathematical behavior, statistical behavior, and market behavior.

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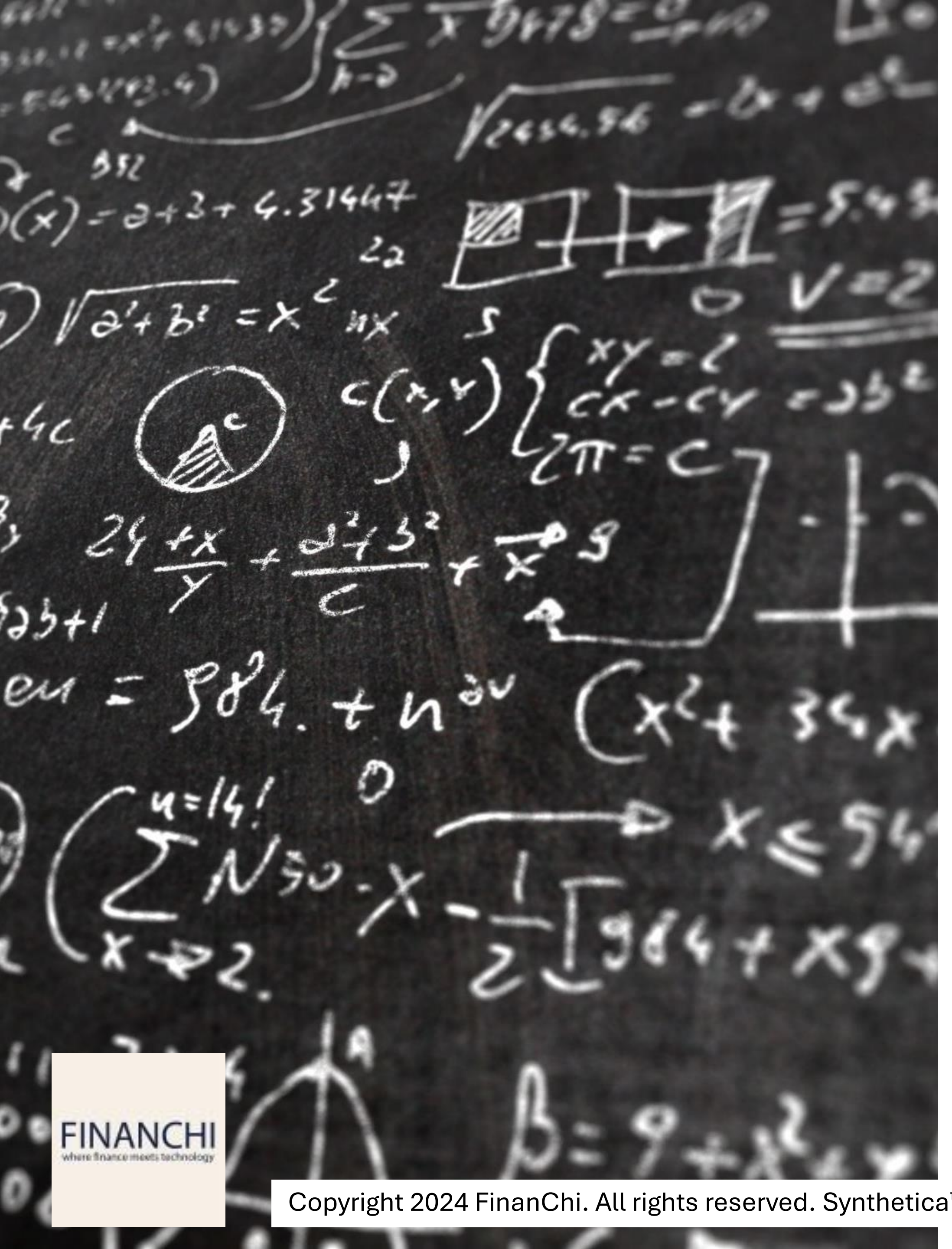

Basic Criteria

Since no closed form solution exists for many measures, depending on the definition of what inputs are used, we want any measure to behave well, in the sense that the functional behavior is sensible, the measurement behavior is sensible, and it is notationally defensible.

From a statistical perspective, market measures must behave similarly when used to measure the same or similar markets.

Finally, the general implications of the efficient market hypothesis must not be violated.

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Definition of a Market Measure

We say a measure M is a measure of market return in some given market if the following 8 constraints hold true.

A review of each constraint and examples follow.

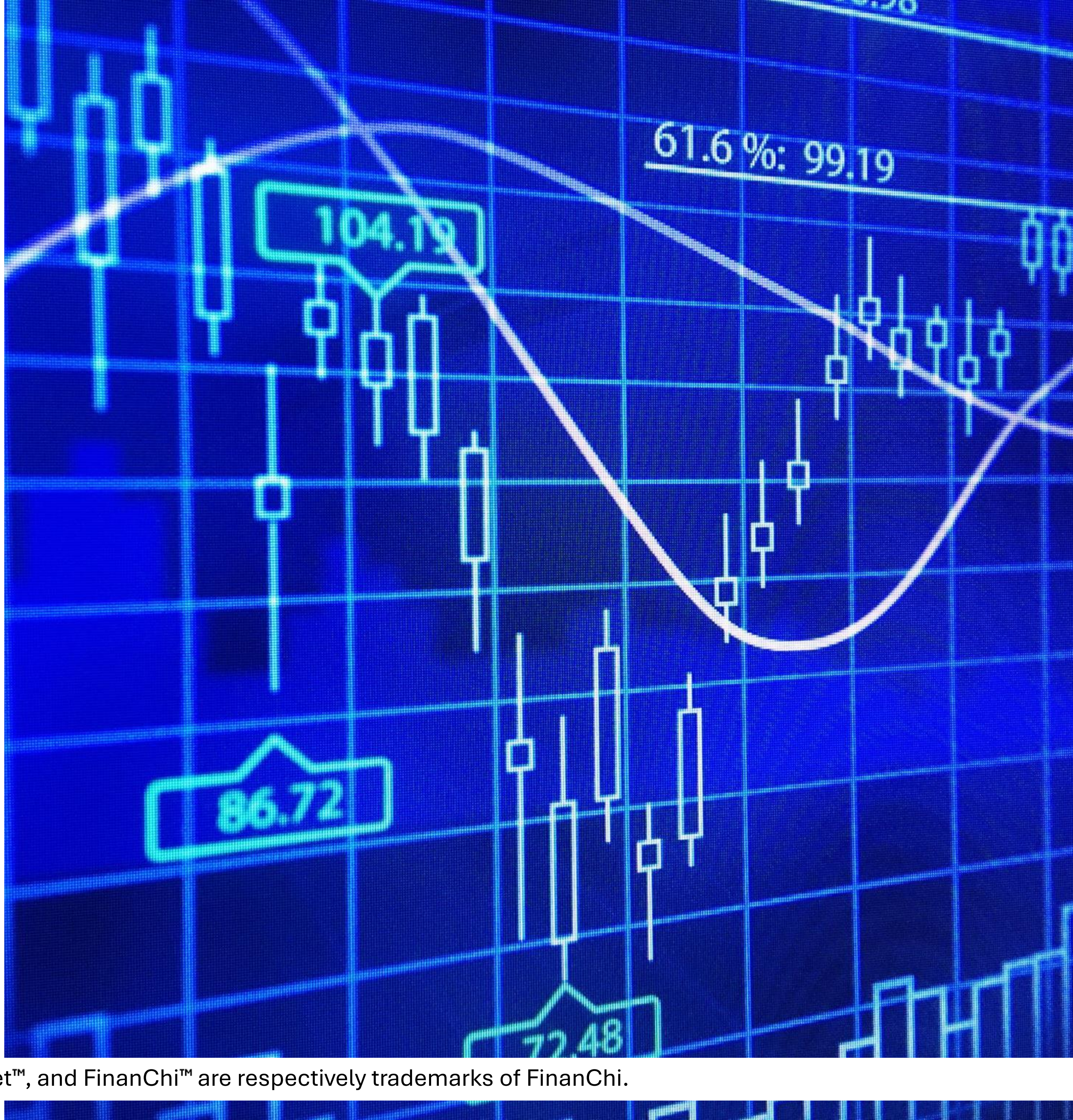
#1 Normality

We assume that investors only use two moments, mean and standard deviation, when evaluating investments. Thus, we have two immediate additional constraints.

#2 Consistent Means

If N is also a measure of market return for the given market, then

M – N = X, where X is a random variable with mean 0, when M and N are sampled over time.



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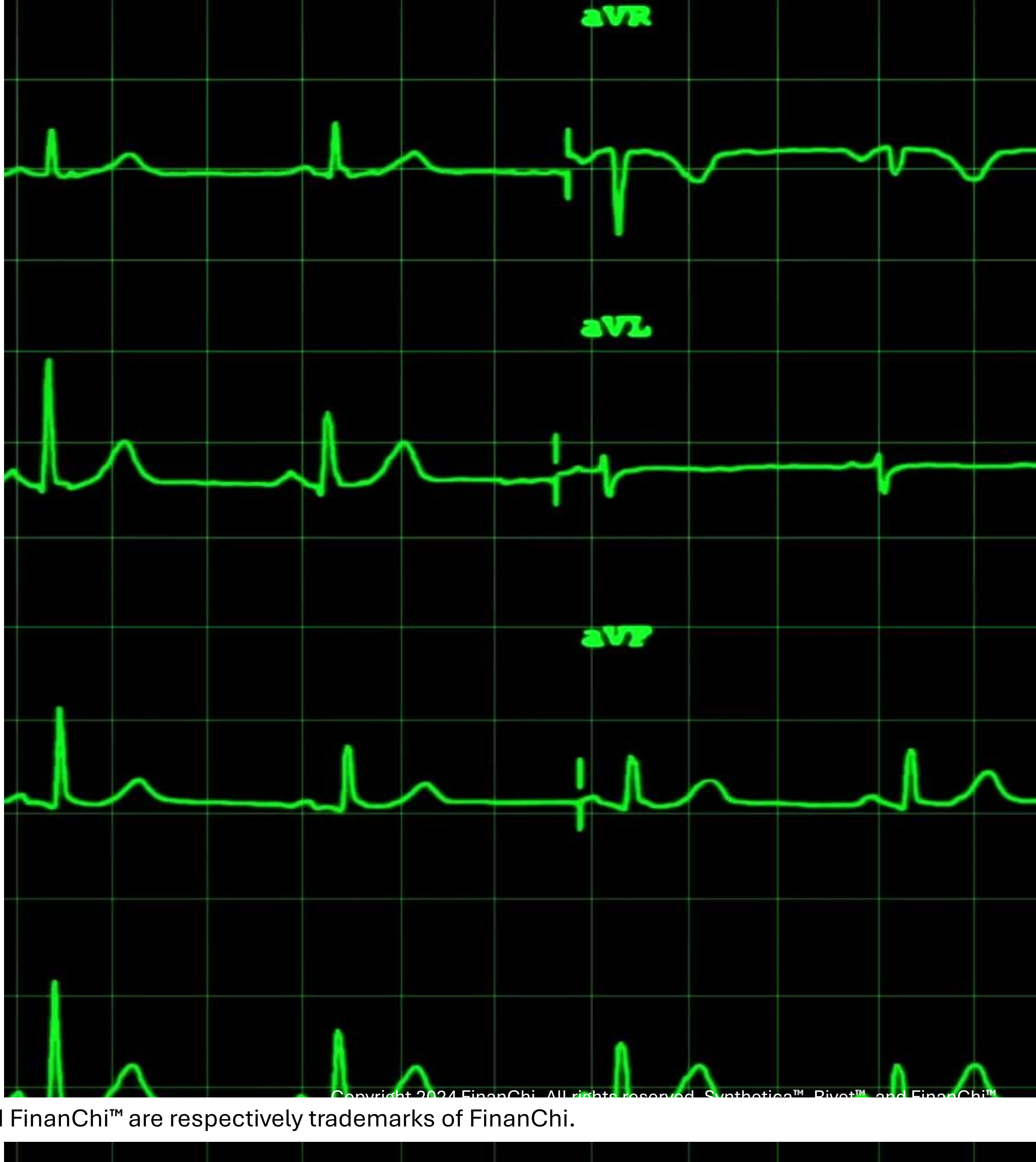
#3 Consistent Volatility

If s(M) and s(N) are the standard deviations of M and N, then

s(M) – s(N) = Y, where Y is a random variable with mean 0, when s(M) and s(N) are sampled over time.



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#4 Continuity

- A hypothesis test that the measure is continuous cannot be rejected.
- Continuity is defined as the meeting the standard limit formulation of the first derivative of a function.

#5 Finite Additivity

Given the measure M on a set (market) X, and a set of subsets with scalar values representing weights of those subsets (summing to 1 over the entire market), there exists epsilon greater than 0, such that if A, B, and A U B are such subsets of X, The hypothesis that

> $wM(A \cup B) - [xM(A) + yM(B)] < \text{epsilon} \Leftrightarrow$ A and B are disjoint, with w, x, and y being the respective scalar weights of A U B, A, and B,

cannot be rejected.

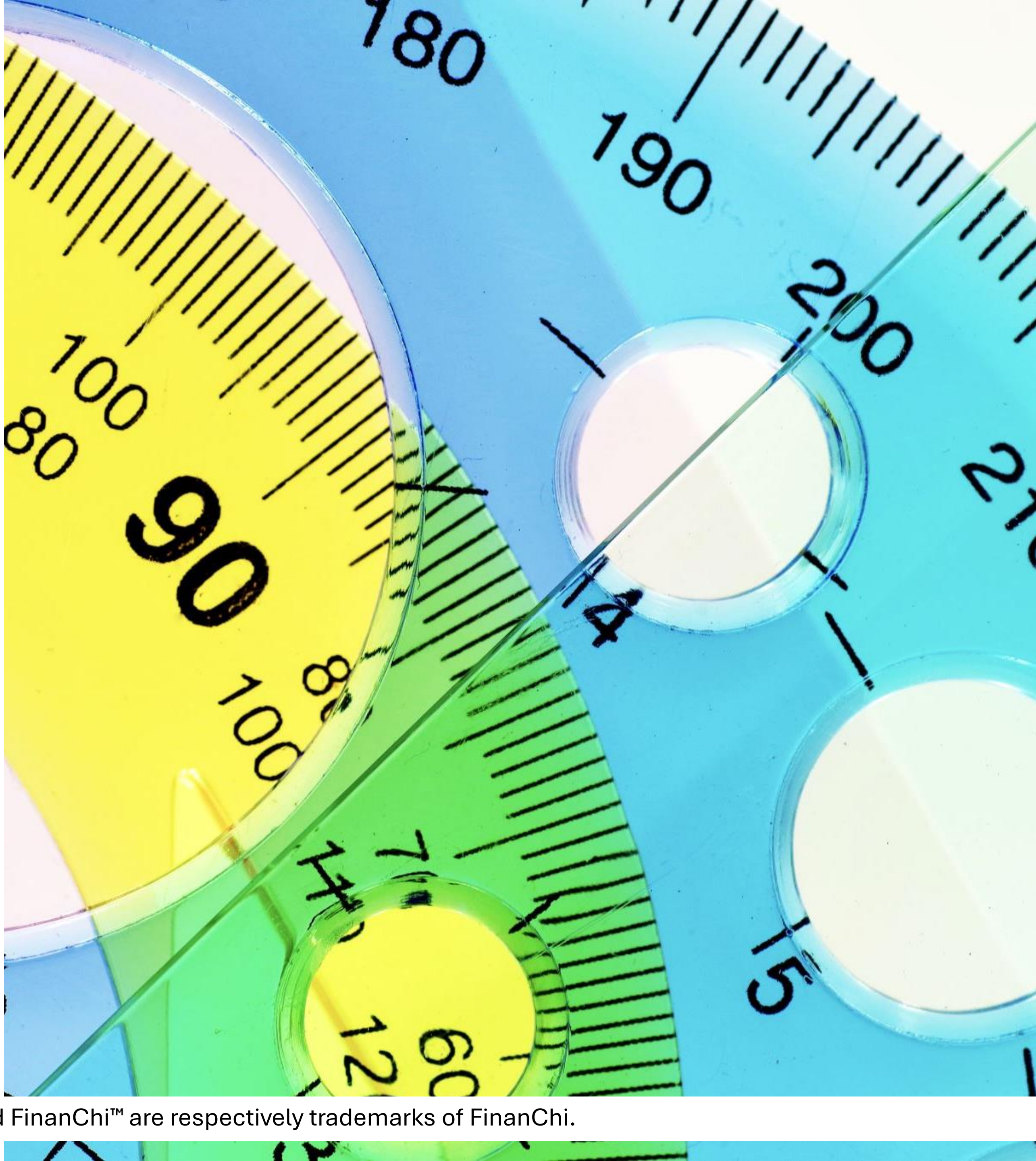
#6 Parity

M is arbitrage-resistant, meaning that if M and N are measured in different ways, and consistently produce different results, then they must eventually converge.

Specifically, M and N must exhibit parity in the traditional sense.

#7 Transitivity

If A, B, and C, are market measures, and the inclusion of A in a set of measures for a market M implies the inclusion of B, and the inclusion of B implies the inclusion of C, then the inclusion of A implies the inclusion of C.



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#8 Inclusion

The S&P 500 is a market measure for the United States.

That is, the set of market measures for the United States is not the empty set.

Additionally, the generally accepted major market measure for each market is considered to be included in the set of measures for the respective market.

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Discussion of the Constraints

While normality is not certain in measuring market returns (Constraint #1), it has been a useful assumption in analyzing past returns and predicting future returns.

One of the major arguments against normality is that large price swings statistically occur more often than expected, which could also make assumptions of underlying continuity (#4) and additivity (#5) difficult to gauge.

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Normality

Since normality is required or implied by many academic and professional analyses, we include it here.

Specifically, we assume normality and the belief that investors, including us, primarily measure market behavior using the mean (#2) and standard deviation (#3) of returns.

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Efficient Markets

The efficient market hypothesis, while not universally accepted, implies that some market behavior must be dependable (#6).

Finally, we must assume that at least one practical measure of market returns does, in fact, exist (#8), or we would have nothing to measure.

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Sparse Data

This brings up one final problem:

is there a loophole in comparing a number of measures to determine if they all are market measures for one market?

That is, we believe that the given constraints are the correct strength to allow for transitivity (#7).

If the S&P 500 is included, implying the Wilshire 5000 and S&P 1000 are included, must it follow logically that we include the S&P 1000 as a result of the inclusion of the Wilshire 5000?

If it is not true, then is it a sparse data problem or an error in constraint definition?

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More on Normality

- The implication of constraint #1 is merely that we assume normality when selecting analytical tools and applying formulas.
- If a formula is only valid for normal populations, we assume it is valid for analyzing market returns, whether or not it is proven, or even true.

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More on Behavior

- For constraints #2 and #3, it is clear that reasonable measures of market return will vary in their means and standard deviations, but we can assume that if the differences remain constant over time using largely the same inputs, a simple translation of the formula is enough to make two measures have the same mean and standard deviation.
- For example, the returns of the Wilshire 5000 are widely regarded as a measure of market returns, yet differ from the returns of the S&P 500 by 47 basis points over time.
- By subtracting 47 basis points from each W5000 data point, the two measures become equivalent in the first moment. Similarly, any difference in the second moment can be cured.

More on Continuity

- Constraint #4 is necessary to reject odd measurements that may vary too much over small periods of time. For example, the function that takes on the value 1 for all irrational numbers in the interval (0,1) and 0 for all rationals, is discontinuous everywhere, but has an average value of 1 over the interval.
- If you estimate the derivative of a measure of returns with respect to time, it may occasionally exhibit discontinuities, but we require that in general (for some $p > 0$) a hypothesis test that the limit of the sample values of the derivative of the measure with respect to time tends to zero as time differences tend to zero should not be rejected at some reasonable p-value.

More on Additivity

- Constraint #5 is required for data that may appear to make sense over some subset of a market. Again, since there will usually not be a closed-form solution for a market measure, it might not be defined or might produce unintuitive values over some set of the market.
- As such, we introduce finite additivity, to allow a measure's behavior over its market to be tested. While it is possible that returns across sectors are related, it is obvious that the weighted sum of sector returns in the S&P 500 must equal the S&P 500 returns.
- In other cases, the formula for a market measure may not correspond easily to subsets of the market and we require a reasonable hypothesis test of finite additivity not be rejected.

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More on Parity

Constraint #6 requires that the measurement not violate the principle of parity.

If the measurement is defined in such a way as to systematically misprice certain financial instruments, then this constraint could be violated (see the example of ADRs of French stocks).

For example, if there were a price control on oil in one market, yet not in another, then an arbitrage opportunity might exist. In this case, measuring the returns in one market using the currency in the first market versus the currency in the second market, using exchange rates might produce different values over a long period. If this is curable through applying constraints #2 and #3 then it may not be a problem.

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Proper Formulaic Definitions

Constraint #7 is the common sense check on the entire set of constraints. If we include one measure A in a set of measures for a market, then all other valid (according to the constraints) measures must also be included when comparing them to A using constraints #2 and #3 and testing against constraints #1 and #4 - #6.

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All Measures are Wrong, but Some are Useful

Finally, for constraint #8, we consider the possibility that a reasonable set of constraints by themselves may allow for measures that are too lenient or useless (not many people would use a market measure that used arbitrarily large or small numbers), and we simply force the inclusion of major market measures.

In the US, we say that the S&P 500 is a valid market measure. This forces all other market measures in the US to be compared to the S&P 500 and each other.

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Further Theoretical Concerns

It is possible that these constraints are not enough to establish a useful market measure for an arbitrary market, and a mathematical proof of sufficiency would be of interest.

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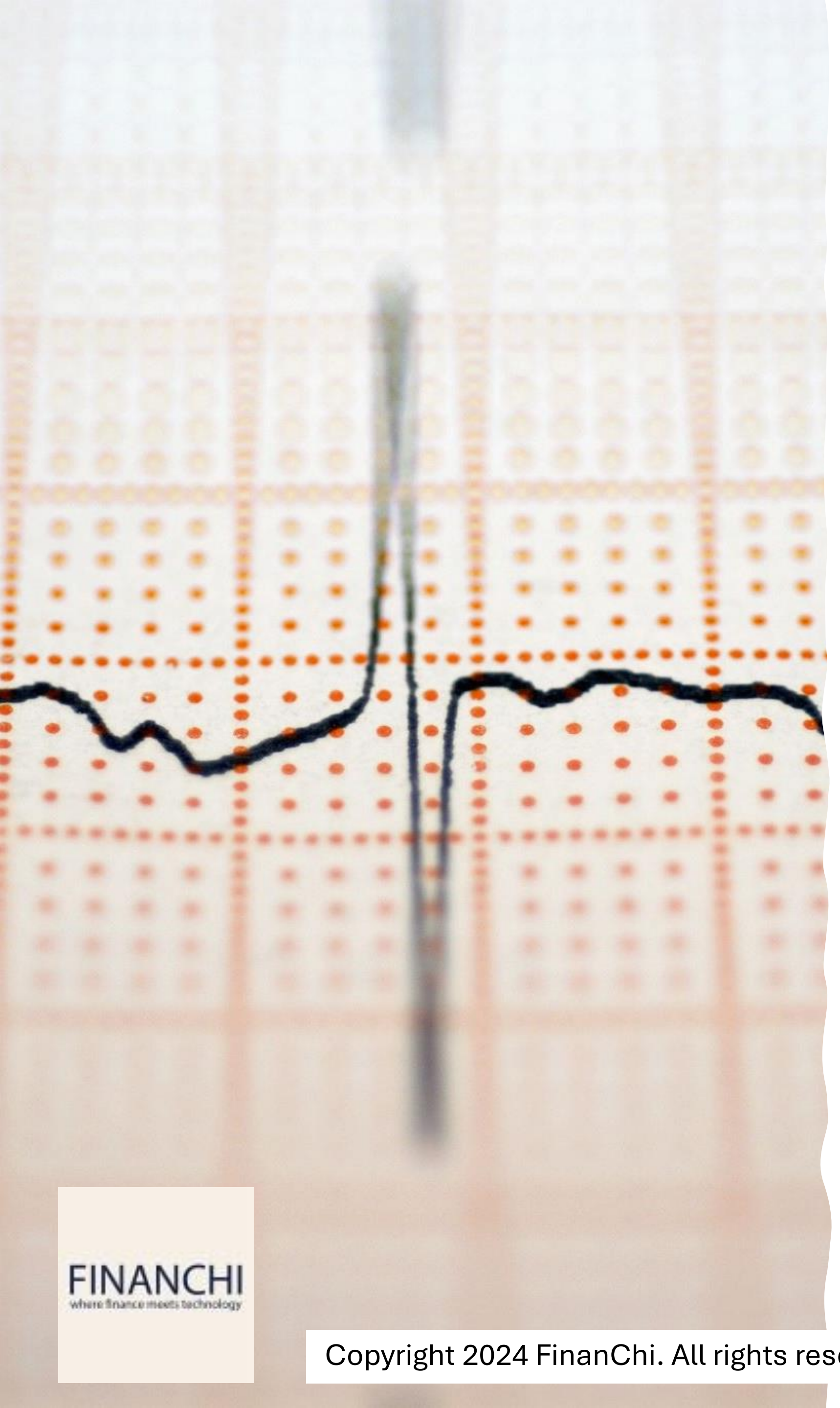

Examples

The following examples are provided for clarity

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The S&P 500

Including the S&P 500 explicitly in measures of US market returns means we don't have to test all 8 criteria. In particular, #1, #2, #3, and clearly #8 need not be tested.

Of interest, a hypothesis test for criterion #4 would provide an implied maximum level of uncertainty that hypothesis tests for other measures should meet.

That is, if the S&P 500 returns appear to model a continuous function under some hypothesis, then other measures of returns for the US should not face a higher bar for a continuity test.

Transitivity (#7) need not be tested, and we can quickly see why Finite Additivity (#5) probably works, as mentioned above, by seeing that the S&P 500 index is the weighted sum of its sector indexes.

Parity (#6) is already broadly believed to constraint most major market indexes like the S&P 500.

Wilshire 5000

Wilshire 5000 returns have been shown to have a stable mean and variance, which can be accounted for by subtraction, when comparing to the S&P 500.

It otherwise manifests all the properties of the S&P 500 with respect to our constraints.

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CAC (Euronext Paris)

If the CAC (top 40 stocks on the French stock exchange) can be considered a measure of market return for the French market or economy, then it meets the criteria on the same basis that the S&P 500 meets the criteria for the US.

An interesting test would be to take the subset of the CAC that is also traded in the US and compare the mean and volatility of returns to the CAC itself.

This would provide a test of constraint #6 (Parity) for the major markets in both the US and France.

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Conclusion

Measuring market returns in different ways allows a wealth of possibilities for analytics.

A rigorous definition of such a measure is required to eliminate many errors in modeling markets.

Synthetica offers a custom measure of market beta, which is based on a proprietary definition of market return.

We use the 8 constraints in this paper to validate any internal measure of returns and factors that logically depend on such a definition.

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Get in Touch

Reach out to the Synthetica team at FinanChi to find out more about our approach and capabilities.